

3.1 average and instantaneous velocity

2014-10-01 day 26

Which of the following is true?

- 1) Average speed is always greater than (or equal to) average velocity.
- 2) Average velocity is always greater than (or equal to) average speed.
- 3) It depends on each situation.

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winner III

a) give me a situation where average speed is greater than average velocity.

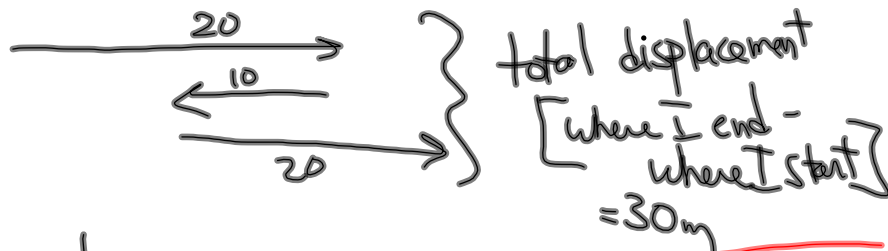
Also

b) give me a situation where average velocity is greater than average speed.

if you voted non-3, give a reason/argument.

winner III

a) give me a situation where
 $\frac{\text{distance}}{\text{time}}$ vs $\frac{\text{displacement}}{\text{time}}$
 average speed is greater than
 average velocity.



total distance
 [all the steps I traveled]
 $= 20 + 10 + 20 = 50$

average speed =
 $\frac{\text{total distance}}{\text{time endured}}$

Situation 2

I go backwards
10 mi. in 10 hrs.

avg velocity = -1 mph
 avg speed = 1 mph

T or W:

Avg speed is always
 non-negative.

True because

distance is always non-negative.

displacement is negative, positive,
 or zero.

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Winner III

Also

b) give me a situation where
average velocity is greater than
average speed.

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$$\text{Average velocity} = \frac{\text{total displacement}}{\text{time endured}}$$

displacement: → determined by how I measure
* units
- distance

slope formula
difference quotient

$$\frac{s(\text{end}) - s(\text{beginning})}{\text{end time} - \text{beginning time}}$$

* direction
- positive or negative
- convention

→ requires a start "place" and an ending place

→ VELOCITY OVER AN INTERVAL

instantaneous velocity - glance at the speedometer

→ velocity AT A POINT

→ challenge is that the idea of velocity is a "2 point" idea

idea 1: approximate instantaneous velocity with average velocity over a narrow relevant interval.

t	1	3	5
s(t)	0	4	14

approx inst. vel
at $t=4$
 $\frac{s(5)-s(3)}{5-3} = \frac{14-4}{2} = 5$

approx inst. vel = 2

idea 2: take the limit of average velocity as the "narrow-ness" shrinks to 0

$$\lim_{t \rightarrow a} \frac{s(t) - s(a)}{t - a}$$

avg velocity

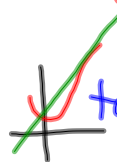
→ implication

I have a fⁿ: $s(t)$ that tells me as many values as "I need"

inst vel at $t=a$ is

idea 3 "geometric" idea

on a graph, the instantaneous velocity will be the slope of the tangent line, (to the curve at that single point of interest)



tangent line is unique

* most closely matches the curve at that point.

* "near" my point, touches the curve in 1 pt

window

$$x: [-5, 5]$$

$$y: [0, 25]$$

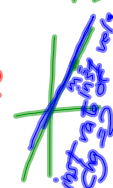
$$y_1 = x^2$$

interested in
 $x=2, y=4$

graph
 $y = 2x$
inst. vel.
at $t=3$

draw tangent

Eqn of tangent line
 $y = 4x + 4$



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$$\begin{aligned} & \text{? } s(t) = 2t \\ & \quad s(a) = 2a \\ \text{inst at } t=a: & \lim_{t \rightarrow a} \frac{s(t) - s(a)}{t - a} \\ \text{vel} & \\ & = \lim_{t \rightarrow a} \frac{2t - 2a}{t - a} = \lim_{t \rightarrow a} \frac{2(t-a)}{t-a} \\ & = \lim_{t \rightarrow a} 2 = 2 \end{aligned}$$